Unit II Knowledge and Reasoning

**Representing Simple Facts in Propositional and Predicate Logic**

**Proportions:**

Proportions are similar to arithmetic expressions. There are operands, which represent the values T or F (True or False) and operators (example, and, or instead of \*, +) and parenthesis are used to aid in determining order of evaluation.

**Proportions are formed accounting to the following rules:**

1. T and F are propositions
2. An identifier is a proportion (An identifier is a sequence of one or more digits and letters, the first of which is a letter).
3. If b is a proportion, then (also a proposition.
4. If b and c are proportions, then are also proportions.

**Examples:**

**The following are proportions:**

F,,, (

**The following are not proportions:**

FF,

**The following are the operators used in proportions:**

Negation:

Conjunction:

Disjunction:

Implication:

Equality:

There exists:

**Transforming English to Propositional Form**

**Example 1**

Consider the following sentences and their corresponding identifiers. Write proportions.

Sentence identifier

It rains

Tour is cancelled

Be wet

Stay at home

1. If it rains but I stay at home, I won’t be wet.

1. I will be wet if it rains.
2. If it rains and the tour is not cancelled or I don’t stay at home, I will be wet.

Or

1. Whether or not the tour is cancelled, I am staying home if it rains

This reduces to

1. Either it does not rain or I am staying home.

**Example 2**

1. It is raining raining
2. It is sunny sunny
3. If it is raining then it is not sunny

**To represent the obvious fact stated by the classical sentence.**

ram is a man.

dhanapal is a man.

We could draw any conclusion about similarities between ram and dhanapal.

man(ram) man(dhanapal)

“The structure of the representation reflects the structure of the knowledge itself”.

To differentiate the relationship between any individual sentences, we need **variables and quantifiers**.

The two quantifier symbols are and, where means for some x or there is an x and means for all x.

Let us explore the use of first order predicate logic as a way of representing knowledge by considering below examples.

**Example 1**

Translate the following sentences into formulae in predicate logic.

1. Venkatesh only likes easy courses.
2. Arts curses are not easy.
3. All the courses in the science department are easy.
4. MCA is a science course.

**Predicate Logic**

1. : easy\_courses(x) likes (Venkatesh, courses(x))
2. : arts\_course(x) easy\_courses(x)
3. : course(x) (science\_dept) easy\_course(x)
4. science\_course(MCA)

**Example 2**

Consider the following facts:

1. Karate fighters are very strong.
2. Mani is a karate fighter.
3. Mani broke some body part of every other karate fighter.
4. Mani broke the leg of the karate fighter who broke the jaw of Raja.
5. Raja is a boxer.
6. Boxers are not as strong as karate fighters.

Represent these facts in predicate logic.

**Predicate Logic**

1. : very\_strong(karate\_fighter(x))
2. Karate\_fighter(Mani)
3. broke(Mani,x) (karate\_fighter(y), bodypart(x))
4. broke(Mani,karate\_fighter\_leg) broke (karate\_fighter\_jaw (Raja)
5. boxer(Raja)
6. strongas (boxer(x), karate\_fighter(y))

**Example 3**

Represent the following facts in Predicate Logic

1. Marcus was a man.
2. Marcus was a Pompeian.
3. All Pompeian’s were Romans.
4. Caesar was a ruler.
5. All Romans were either loyal to Caesar or hated him.
6. Everyone is loyal to someone.
7. People only try to assassinate rulers they are not loyal to.
8. Marcus tried to assassinate Caesar.

**Predicate Logic**

1. Man(Marcus)
2. Pompeian(Marcus)
3. Pompeian(x) Roman(x)
4. Ruler (Caesar)
5. Roman(x) loyalto(x, Caesar) hate (x,Caesar)
6. loyalto(x,y)
7. person(x) ruler(y) tryassassinate (x,y) loyalto(x,y)
8. Tryassassinate (Marcus, Caesar)

**Example 4**

Translate the following sentence into formulae in Predicate Logic.

1. Vishnu likes all kinds of fruits.
2. Mani is a good student.
3. All good students have high grades.
4. All students with high grades are bright.
5. Narayanan likes seeing English movies.

**Predicate Logic**

1. fruits(x) likes(Vishnu,x)
2. Goodstudent (Mani)
3. goodstudent highgrades(x)
4. student (x) highgrade(x) bright (x)
5. Englishmovie (x) likes(Narayanan, x)

**RESOLUTION**

* Resolution is a powerful technique for reducing the proof process to the execution of a simple mechanical task.
* Resolution produces proofs by refutation. That is to prove a statement, resolution attempts to show that the negation of the statement produces a contradiction with the known statements.

**Conversion to Clausal Form**

1. Resolution requires that all statements be converted to a clausal form. We define a clause as the disjunction of a number of literals.
2. A ground clause is one in which no variables occur in the expression.
3. A horn clause is a clause with at most one positive literal.

**Algorithm: Clausal Conversion**

1. Eliminate all implication ( ) using the fact that is equivalent to
2. Move all negations into immediately precede a single term, using the fact , Demorgan’s laws, and the standard correspondences between quantifiers ,

( and

1. Rename variables if necessary so that all remaining quantifiers have different variable assignments.

For example, in the expression rename the sec and ‘dummy’ variable x which is bound by the existential quantifier to be a different variable, say y, to give

1. Replace existentially quantified variables with special functions and eliminate the corresponding quantifiers.

For example, can be transformed into sum (d)

Where d is a function with no arguments that somehow produces a value that satisfies sum and these generated functions are called skolem functions.

1. Move all universal quantifiers to the left of the expression, that is, disjunctions are moved down to literals.
2. Eliminate all universal quantifiers and conjunctions since they are retained implicitly. The resulting expressions are clauses and the set of such expressions is said to be in clausal form.

**RESOLUTION IN PROPORTIONAL LOGIC**

* The best way to obtain a general idea of the resolution inference rule is to understand how it applies to ground clauses.
* Suppose we have two ground clauses p1,p2,………pn and q1,q2,……….qm.
* We assume that all of the pi and qj are distinct.
* Note that one of these clauses contains a literal that is the exact negation of one of the literals in the other clause.
* From these two parent clauses we can infer a new clause.
* From these two parent clauses we can infer a new clause called the resolvant of the two.
* The resolvant if computed by taking the disjunction of the two clauses and then eliminating complementary pair p1,

Example

Consider the following proportions.

1. p
2. t

Convert these into clause form. Using resolution prove that r is true.

Clause form

1. p

To prove r is true, we assume that is true and ultimately we arrive at a contradiction.

Resolution proof

cannot be true. So, we arrive at a contradiction. Our assumption is wrong. That is

true is wrong. That is r is true.

RESOLUTION IN PREDICATE LOGIC

Resolution is the process of establishing the truths by “Proof by refutation” technique.

**Example 1**

Consider the following facts.

1. Mani only likes easy games.
2. Boxing is hard.
3. All the indoor games are easy.
4. Table tennis is an indoor game.
5. Represent these facts in Predicate Logic.
6. From the above facts, use resolution to deduce that, Mani likes Table Tennis.
7. **Predicate Logic**

10. **Clause form**

To prove Mani likes Table tennis, let us assume that Mani does not like Table tennis.

Resolution by refutation



What we have assumed here becomes false. So we conclude that Mani likes Table Tennis.

**Example 2**

Consider the following facts.

1. Ravi only like easy courses.
2. Science courses are hard.
3. All the courses in the Engineering department are easy.
4. E103 is an Engineering course.

Use resolution to answer the question, what course would Ravi like?

**Predicate Logic**

**Clause form**

**Resolution**

In order to find “What course Ravi likes?” we have to check whether Ravi like any course?

To answer this, we attempt to show that likes (Ravi, t) produce a contradiction.

If a contradiction arises then we come to a conclusion that “Ravi likes a course” and then we will find what course it is.



Therefore, the answer to the question can now be derived from the chain of unification that leads back to starting stage. Let us now find the exact course, which Ravi likes by appending the new expression **likes (Ravi, t)**

Thus, **Ravi likes E103** is proved.

**Example 3:**

Consider the following sentences.

1. Narayanan likes all kinds of food.
2. Apples are food.
3. Chicken is food.
4. Anything any one eats and is not killed by is food.
5. Ram eats peanuts and is still alive.
6. Sita eats everything Ram eats.
7. Translate these sentences into formulae in predicate logic.
8. Convert the formulae of part (a) into clause form.
9. Prove that Ram liked peanuts using resolution.
10. **Predicate Logic:**
11. **Clause form:**

(or)

1. **Resolution:**

With the given statements it is not possible to prove “Ram likes peanuts”. So let us introduce one more statement as follows.

1. Ram likes whatever Narayanan likes.

Predicate Logic:

Clause form:

To prove “Ram likes peanuts”, let us assume that “Ram does not like peanuts”.



Thus we obtained contradiction. Hence, the assumption that we made is wrong. That is “Ram doesn’t like peanuts” is wrong. Hence we proved that “Ram likes peanuts”.

**Example 4:**

Consider the following facts.

1. The members of Goodwill club are Raja and Ravi.
2. Raja is married to Rani.
3. Ravi is Rani’s brother.
4. Represent these facts in predicate logic.
5. From the above facts, use resolution to deduce that “Ravi is not married”.
6. **Predicate Logic:**
7. **Clause form:**

Note that with the given facts it is not possible to deduce that “Ravi is not married” using resolution. Hence we add one more fact as follows.

1. If a person is brother to somebody then he cannot marry that person.

**Predicate logic:**

**Clause form:**

To prove “Ravi is not married”, let us assume “Ravi is married”.

**Resolution proof:**



Thus we obtained contradiction. Hence, the assumption that we made is wrong. That is “Ravi is married” is wrong. Hence we proved that “Ravi is not married”.